



Kaufman - Trading Systems & Methods -

Appendix 4

VISIT...

LANZAROTE
Caliente.COM

APPENDIX 4

Trigonometric Regression for Finding Cycles

The following computer programs and examples solve the cycle problems in Chapter 8.

SINGLE-FREQUENCY TRIGONOMETRIC REGRESSION

The FORTRAN program *TRIG1* and the subroutine *LINREG* are used to find the single-frequency representation of the copper cycle. The program output is clearly separated into the following information:

1. Input data, where each time period is the average cash price for a calendar quarter.
2. The solution to the linear regression, giving the detrending line. With $b = .267$, there is an inflationary bias of $+.267\%$ per quarter.
3. The detrended data resulting from subtracting the line values (2) from the original data (1).
4. Intermediate values for α , ω , and T .
5. The constant values a and b for the normal equations solving the single-frequency problem.
6. The cycle resulting from the detrended data.
7. The final cycle with the trend added back.

The results show a copper cycle of approximately 8.4 quarters, or slightly more than 2½ years.

An additional test was run on monthly cash corn prices from 1964 through 1983 to see if the seasonal cycle dominated the detrended pattern. The linear regression equation used for detrending was calculated as:

$$y = .939 + .01x$$

showing only a 1¢ per bushel per month rate of inflation, despite the bull markets in 1973 and 1980 through 1981. The cycle showed a period of 21.4 months, with the last highs in the cycle in August 1983 and the last lows in September 1982. Because this is clearly not a seasonal pattern, it must be either:

1. Dominated by other supply-demand characteristics, such as stocks, or.
2. Distorted by the nonseasonal rallies of 1973 and 1980, which each took three years to return to the traditional seasonal pattern.

```
PROGRAM TRIG1
C---- Single Frequency Trigonometric Regression
C---- Copyright 1986 PJ Kaufman

DIMENSION X(250), Y(250), D(250), R(250)
DOUBLE PRECISION SC2,SCD

DATA MAX/250/

OPEN(6,FILE='PRN')

C---- Read data, set X to incremental time
I = 1
WRITE(*,7000)
```

```

7000 FORMAT(' SINGLE FREQUENCY TRIGONOMETRIC REGRESSION'/
+         ' Enter data 1 per line//' Extra <return> ends input '//
+         ' Enter below:')

10 WRITE(*,7001)I
7001 FORMAT(14,'>'\)
READ(*,5000)Y(I)
5000 FORMAT(BN,F8.2)
X(I) = I
IF(Y(I).NE.0)THEN
    IF(I.GT.MAX)STOP 'Maximum data exceeded'
    I = I+1
    GOTO 10
ENDIF
N = I-1
WRITE(6,6000)
6000 FORMAT('1Single Frequency Trigonometric Regression',20X,
+         'Data Input'///' Time',16X,'Prices')
DO 40 J = 1,N,4
40     WRITE(6,6001)J,(Y(I),I=J,J+3)
6001     FORMAT(14,4F8.2)

C---- Linear regression analysis for detrending
CALL LINREG(N,Y,A,B,SD)
WRITE(6,6002)A,B
6002 FORMAT(/'Linear regression results: A =',F8.3,', B =',F8.3)
C---- Detrend data into D and computer sums for equation (4)
DO 60 I = 1,N
60     D(I) = Y(I) - (A+B*I)
C---- Print detrended data
WRITE(6,6003)
6003 FORMAT(/' Detrended data'/)
DO 65 I = 1,N,4
65     WRITE(6,6001)I,(D(J),J=I,I+3)
    SC2 = 0
    SCD = 0
C---- Solve equation (4) using detrended data
DO 70 I = 2,N-1
    DI = D(I)
    SC2 = SC2 + DI*DI
    SCD = SCD + DI*(D(I-1)+D(I+1))
70

    ALPHA = SCD/SC2
    WRITE(6,6004)SC2,SCD,ALPHA
6004 FORMAT(/'Sum C-squared =',F8.1,', Sum C x D =',F8.1,', Alpha =',
+         F8.3)
C---- Solve for omega
W = ACOS(ALPHA/2)
T = 360/W
WRITE(6,6014)W,T
6014 FORMAT(/'Omega (W) =',F5.1,' degrees, Period (T) =',F6.2,
+         'time units')

C---- Sums for normal equations
COS2 = 0
COSSIN = 0
YCOS = 0
SINCOS = 0
SIN2 = 0
YSIN = 0

```

```

      DO 80 I = 1,N
        C = COS(W*I)
        S = SIN(W*I)
        COS2 = COS2 + C*C
        COSSIN = COSSIN + C*S
        YCOS = YCOS + Y(I)*C
        SIN COS = SIN COS + S*C
        SIN2 = SIN2 + S*S
80      YSIN = YSIN + Y(I)*S

C---- Solve normal equations
TB = (YSIN*COS2 - YCOS)/(SIN2*COS2 - COSSIN)
TA = (Y COS - B*COSSIN)/COS2
WRITE(6,6005)TA,TB
6005 FORMAT('/ Solution to normal equations: A =',F8.3,', B =',F8.3)

C---- Values of fitted curve using detrended data
DO 90 I = 1,N
  90   R(I) = TA*COS(W*I) + TB*SIN(W*I)
      WRITE(6,6006)
6006  FORMAT('/ Trigonometric regression results using detrended data',
+         /)
      DO 100 I = 1,N,4
100    WRITE(6,6001)I,(R(J),J=I,I+3)

C---- Add trend back to result
DO 110 I = 1,N
110   R(I) = R(I) + A + B*I

      WRITE(6,6007)
6007  FORMAT('/ Final regression results with trend added back'/)
DO 120 I = 1,N,4
120   WRITE(6,6001)I, (R(J),J=I,I+3)

      CALL EXIT
      END

SUBROUTINE LINREG(N,DATA,A,B,SD)
C---- Generalized simple linear regression
      DIMENSION DATA(2)

C---- Initialize sums
      SX=0.
      SY=0.
      SXY=0.
      SX2=0.
      A=0.
      B=0.
      SD=0.
      IF(N.LE.2)RETURN

DO 100 I=2,N
  X = I
  Y = DATA(I)
  SX=SX+X
  SY=SY+Y
  SXY=SXY+Y*X
  SX2=SX2+X*X
100  CONTINUE

M=N-1
B=(M*SXY-SX*SY) / (M*SX2-SX*SX)
A=(SY-B*SX) / M

```

```

C----- Residuals
SSR=0
DO 200 I=2,N
  Y=DATA(I)
  R=Y-(A+B*I)
  SSR=SSR+R*R
200  CONTINUE

SD=SQRT(SSR/M)
RETURN
END

```

Single Frequency Trigonometric Regression

Time	Prices			
1	22.12	22.46	22.17	22.00
5	23.18	24.56	25.57	30.59
9	28.23	33.77	35.90	40.05
13	46.22	51.48	40.76	40.16
17	36.51	29.30	30.36	36.42
21	39.75	30.07	29.08	32.13
25	38.94	42.95	43.38	46.23
29	47.70	46.98	35.78	27.35
33	25.40	29.45	27.15	28.48
37	32.74	33.53	30.01	29.25
41	36.82	45.07	55.13	65.51
45	66.56	70.06	27.30	35.62
49	32.06	31.46	35.75	36.46
53	38.22	43.24	45.46	38.96
57	37.08	38.72	34.01	33.00
61	35.07	40.23	41.63	44.95
65	51.12	63.71	59.56	63.38

Linear regression results: A = 28.889, B = .267

Detrended data

1	-7.04	-6.96	-7.52	-7.96
5	-7.05	-5.93	-5.19	-1.44
9	-3.06	2.21	4.07	7.95
13	13.86	18.85	7.86	7.00
17	3.08	-4.40	-3.61	2.19
21	5.25	-4.70	-5.96	-3.17
25	3.37	7.11	7.28	9.86
29	11.06	10.07	-1.39	-10.09
33	-12.31	-8.52	-11.09	-10.03
37	-6.04	-5.51	-9.30	-10.33
41	-3.02	4.96	14.75	24.86
45	25.65	28.88	-14.15	-6.10
49	-9.92	-10.79	-6.77	-6.32
53	-4.83	-.08	1.87	-4.89
57	-7.04	-5.67	-10.64	-11.92
61	-10.12	-5.23	-4.09	-1.04
65	4.86	17.19	12.77	16.32

Sum C-squared = 6338.4, Sum C x D = 9282.2, Alpha = 1.464

Omega (W) = .7 degrees, Period (T) =480.50 time units

Solution to normal equations: A = -1.603, B = 1.831

Trigonometric regression results using detrended data

1	.81	1.78	1.80	.86
5	-.54	-1.66	-1.88	-1.10
9	.27	1.50	1.92	1.32
13	.01	-1.31	-1.92	-1.51
17	-.28	1.09	1.88	1.66
21	.56	-.85	-1.80	-1.79
25	-.82	.59	1.68	1.87
29	1.06	-.32	-1.53	-1.92
33	-1.28	.04	1.34	1.92
37	1.47	.23	-1.13	-1.89
41	-1.64	-.51	.89	1.82
45	1.77	.77	-.64	-1.71
49	-1.86	-1.02	.37	1.56
53	1.91	1.24	-.09	-1.38
57	-1.93	-1.44	-.19	1.17
61	1.90	1.61	.46	-.94
65	-1.83	-1.75	-.72	.69

Final regression results with trend added back

1	29.96	31.21	31.50	30.82
5	29.68	28.83	28.88	29.93
9	31.57	33.06	33.75	33.41
13	32.37	31.32	30.98	31.66
17	33.15	34.79	35.85	35.90
21	35.06	33.92	33.23	33.52
25	34.75	36.43	37.79	38.24
29	37.70	36.58	35.64	35.52
33	36.43	38.02	39.59	40.43
37	40.25	39.28	38.18	37.69
41	38.21	39.60	41.27	42.46
45	42.68	41.95	40.81	40.01
49	40.17	41.23	42.89	44.34
53	44.96	44.56	43.49	42.47
57	42.19	47.95	44.47	46.09
61	47.09	47.07	46.18	45.05
65	44.42	44.78	46.07	47.74

663

TWO-FREQUENCY TRIGONOMETRIC REGRESSION

The FORTRAN program *TRIG2* and its subroutines *LINREG* (found in Appendix 4) and *MTX* are used to find the two-frequency representation of the copper cycle. The program output is clearly separated into the following steps.

1. Input data, where each time period is the average cash price for a calendar quarter.
2. The solution to the linear regression, giving the detrending line.
3. The detrended data resulting from subtracting the line values (2) from the original data (1).
4. Intermediate values for α_1 , α_2 , ω_1 , and ω_2 .
5. Resulting values a_1 , b_1 , a_2 , and b_2 , which are derived from the matrix solution using Gaussian elimination.
6. The cycle resulting from the detrended data.
7. The final cycle with the trend added back.

```

PROGRAM TRIG2
C---- 2-Frequency Trigonometric Regression
C---- Copyright 1986 PJ Kaufman

DIMENSION X(250), Y(250), D(250), R(250), A(4,4), B(4)
EQUIVALENCE (A(1,1),C2W1),(A(1,2),CW1SW1),(A(1,3),CW1CW2),

```

```

+      (A(1,4),CW1SW2),(A(2,1),DUP1),(A(2,2),S2W1),
+      (A(2,3),SW1CW2),(A(2,4),SW1SW2),(A(3,1),DUP2),
+      (A(3,2),DUP3),(A(3,3),C2W2),(A(3,4),CW2SW2),
+      (A(4,1),DUP4),(A(4,2),DUP5),(A(4,3),DUP6),
+      (A(4,4),S2W2),(B(1),YCW1),(B(2),YSW1),(B(3),YCW2),
+      (B(4),YSW2),(B(1),A1),(B(2),B1),(B(3),A2),(B(4),B2)
DATA MAX/250/,NDIM/4/

OPEN(6,FILE='PRN')

C---- Read data, set X to incremental time
I = 1
WRITE(*,7000)
7000 FORMAT(' 2-FREQUENCY TRIGONOMETRIC REGRESSION'/
+      ' Enter data 1 per line'/'Extra <return> end input'//
+      ' Enter below:')

10 WRITE(*,7001)I
7001 FORMAT(14,'>'\)
READ(*,5000)Y(I)
5000 FORMAT(BN,F8.2)
X(I) = I
IF(Y(I).NE.0)THEN
    IF(I.GT.MAX)STOP 'Maximum data exceeded'
    I = I+1
    GOTO 10
ENDIF
N = I-1
WRITE(6,6000)
6000 FORMAT('12-Frequency Trigonometric Regression'//
+      'Time',16X,'Prices')
DO 40 J = 1,N,4
40    WRITE(6,6001)J,(Y(I),I=J,J+3)
6001    FORMAT(14,4F8.2)

C---- Linear regression analysis for detrending
CALL LINREG(N,Y,ALIN,BLIN,SD)
WRITE(6,6002)ALIN,BLIN
6002 FORMAT(' Linear regression results: A =',F8.3,', B =',F8.3)

C---- Detrend data into D and computer sums for equation (4)
DO 60 I = 1,N
60    D(I) = Y(I) - (ALIN+BLIN*I)

C---- Print detrended data
WRITE(6,6003)
6003 FORMAT(' Detrended data'//)
DO 65 I = 1,N,4
65    WRITE(6,6001)I,(D(J),J=I,I+3)
SC2 = 0
SCD = 0
SCP = 0
SD2 = 0
SDP = 0

C---- Solve for alpha and alpha2 using detrended data
DO 70 I = 2,N-3
C = D(I) + D(I+2)
T = D(I+1)
P = D(I-1) + D(I+3)
SC2 = SC2 + C*C
SCD = SCD + C*T
SCP = SCP + C*P
SD2 = SD2 + T*T

```



```

70   SDP = SDP + T*P
    ALPHA2 = (SDP*SC2-SCP)/(SD2*SC2-SCD)
    ALPHA1 = (SCP-ALPHA2*SCD)/SC2
    T = SQRT(ALPHA1*ALPHA1+8*(1+ALPHA2/2))
    W1 = ACOS((ALPHA1+T)/4)
    W2 = ACOS((ALPHA1-T)/4)

    WRITE(6,6004)SC2,SCD,SCP,SD2,SDP,ALPHA1,ALPHA2,W1,W2
6004 FORMAT(' Intermediate values: '//
+          ' SUMS C2 =', F8.1,', C*D =',F8.1,', C*P =',F8.1/
+          ' D2 =', F8.1,', D*P =',F8.1// ' Alpha1 =',F8.3,
+          ' Alpha2 =', F8.3,' Omegal =',F5.2,', Omega2 =',F5.2)

C---- Sums for normal equations. . .to be used for matrix solution
C2W1 = 0
CW1SW1 = 0
CW1CW2 = 0
CW1SW2 = 0
YCW1 = 0
S2W1 = 0
SW1CW2 = 0
SW1SW2 = 0
YSW1 = 0
C2W2 = 0
CW2SW2 = 0
YCW2 = 0
S2W2 = 0
YSW2 = 0

DO 100 I = 1,N
    D1 = D(I)
    SW1 = SIN(W1*I)
    CW1 = COS(W1*I)
    SW2 = SIN(W2*I)
    CW2 = COS(W2*I)
    C2W1 = C2W1 + CW1*SW1
    CW1SW1 = CW1SW1 + CW1*SW1
    CW1CW2 = CW1CW2 + CW1*CW2
    CW1SW2 = CW1SW2 + CW1*SW2
    YCW1 = YCW1 + D1*CW1
    S2W1 = S2W1 + SW1*SW1
    SW1CW2 = SW1CW2 + SW1*CW2
    SW1SW2 = SW1SW2 + SW1*SW2
    YSW1 = YSW1 + D1
    C2W2 = C2W2 + CW2*CW2
    CW2SW2 = CW2SW2 + CW2*SW2
    YCW2 = YCW2 + D1*CW2
    S2W2 = S2W2 + SW2*SW2
100   YSW2 = YSW2 + D1*SW2
C---- Duplicate calculations for matrix
DUP1 = CW1SW1
DUP2 = CW1CW2
DUP3 = SW1CW2
DUP4 = CW1SW2
DUP5 = SW1SW2
DUP6 = CW2SW2

WRITE(6,6009)
6009 FORMAT(' Coefficient matrix: '//
DO 110 I = 1,NDIM
110   WRITE(6,6010)(A(I,J),J=1,4),B(I)
6010   FORMAT(5F8.3)

```

```

C---- Solve using matrix Gaussian Elimination
CALL MTX(A,B,NDIM)

C---- Solution vector
WRITE(6,6011)(B(I),I=1,NDIM)
6011 FORMAT('/' Solution vector:'/4F8.3)
C---- Values of fitted curve using detrended data
DO 90 I = 1,N
  90   R(I) = A1*COS(W1*I) + B1*SIN(W1*I) + A2*COS(W2*I) +
      + B2*SIN(W2*I)
  WRITE(6,6006)
6006 FORMAT('/' Trigonometric regression results using detrended data',
      + /)
DO 105 I = 1,N,4
  105  WRITE(6,6001)I,(R(J),J=I,I+3)

C---- Add trend back to result
DO 115 I = 1,N
  115  R(I) = R(I) + ALIN + BLIN*I

  WRITE(6,6007)
6007 FORMAT('/' Final results with trend added back'/)
DO 120 I = 1,N,4
  120  WRITE(6,6001)I,(R(J),J=I,I+3)

CALL EXIT
END

SUBROUTINE MTX(A,C,N)
C---- Matrix solution to simultaneous linear equations
C---- Copyright 1986 PJ Kaufman
  DIMENSION A(4,4), C(4), A1(4,4), C1(4)

C---- Process row by row (Gaussian Elimination)
DO 100 I = 1,N
  DIV = A(I,I)

  DO 40 J = 1,N
    40   A(I,J) = A(I,J)/DIV
    C(I) = C(I)/DIV

C---- Zero out column I for each row
DO 60 J = 1,N
  IF(J.EQ.I)GOTO 60
  FACTOR = A(J,I)
  DO 50 K = I,N
    50   A(J,K) = A(J,K) - A(I,K)*FACTOR
    C(J) = C(J) - C(I)*FACTOR
  60   CONTINUE
100  CONTINUE

RETURN
END

```

2-Frequency Trigonometric Regression

Time	Prices			
1	22.12	22.46	22.17	22.00
5	23.18	24.56	25.57	30.59
9	28.23	33.77	35.90	40.05
13	46.22	51.48	40.76	40.16
17	36.51	29.30	30.36	36.42
21	39.75	30.07	29.08	32.13

25	38.94	42.95	43.38	46.23
29	47.70	46.98	35.78	27.35
33	25.40	29.45	27.15	28.48
37	32.74	33.53	30.01	29.25
41	36.82	45.07	55.13	65.51
45	66.56	70.06	27.30	35.62
49	32.06	31.46	35.75	36.46
53	38.22	43.24	45.46	38.96
57	37.08	38.72	34.01	33.00
61	35.07	40.23	41.63	44.95
65	51.12	63.71	59.56	63.38

Linear regression results: A = 28.889, B = .267

Detrended data

1	-7.04	-6.96	-7.52	-7.96
5	-7.05	-5.93	-5.19	-.44
9	-3.06	2.21	4.07	7.95
13	13.86	18.85	7.86	7.00
17	3.08	-4.40	-3.61	2.19
21	5.25	-4.70	-5.96	-3.17
25	3.37	7.11	7.28	9.86
29	11.06	10.07	-1.39	-10.09
33	-12.31	-8.52	-11.09	-10.03
37	-6.04	-5.51	-9.30	-10.33
41	-3.02	4.96	14.75	24.86
45	25.65	28.88	-14.15	-6.10
49	-9.92	-10.79	-6.77	-6.32
53	-4.83	-.08	1.87	-4.89
57	-7.04	-5.67	-10.64	-11.92
61	-10.12	-5.23	-4.09	-1.04
65	4.86	17.19	12.77	16.32

Intermediate values:

SUMS C2 = 17396.7, C*D = 8753.0, C*P = 10475.1
 D2 = 6126.9, D*P = 5499.2

Alpha1 = .151, Alpha2 = .898, Omega1 = .47, Omega2 = 2.52

Coefficient matrix:

34.206	.790	-.461	.426	124.829
.790	33.794	-.454	.479	-7.036
-.461	-.454	33.742	-.781	-33.500
.426	.479	-.781	34.258	28.219

Solution vector:

3.635	-.317	-.930	.762
-------	-------	-------	------

Trigonometric regression results using detrended data

1	4.29	.84	.69	-1.20
5	-3.72	-2.38	-4.57	-2.18
9	-1.06	-.43	3.23	2.12
13	4.21	3.45	1.39	1.82
17	-2.17	-1.93	-3.36	-4.39
21	-1.80	-2.78	.67	1.72
25	2.23	4.76	2.31	3.24
29	1.05	-1.35	-1.09	-4.45
33	-2.90	-3.25	-2.67	.70
37	.12	3.49	3.41	2.97
41	3.91	.27	.59	-2.04
45	-3.65	-2.57	-4.56	-1.50

49	-1.06	.38	3.44	2.19
53	4.54	2.79	1.38	1.20
57	-2.70	-1.91	-3.91	-3.91
61	-1.68	-2.44	1.44	1.69
65	2.86	4.60	2.08	3.19

Final results	with trend	added	back
1	33.44	30.26	30.38
5	26.50	28.12	26.19
9	30.23	31.13	35.05
13	36.58	36.08	34.29
17	31.26	31.77	30.61
21	32.71	31.99	35.70
25	37.80	40.60	38.42
29	37.69	35.56	36.08
33	34.81	34.72	35.57
37	38.89	42.54	42.72
41	43.76	40.38	40.97
45	37.26	38.61	36.89
49	40.93	42.63	45.96
53	47.59	46.11	44.96
57	41.42	42.47	40.75
61	43.50	43.01	47.16
65	49.12	51.13	48.87